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A Vacation Queueing Model with Bulk Service Rule

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Abstract: **The present paper analyzes a vacation queueing model with bulk service rule. Considering distribution of the system occupancy just before and after the departure epochs using discrete-time analysis. The inter-arrival times of customers are assumed to be independent and geometrically distributed. The server works at different rate, rather than completely stopping service vacations. The service times during a service period and vacation time geometric distribution. Some performance measures and a cost model have been presented and an optimum service rate has been obtained using convex function. Numerical results showing the effect of the parameters of the model on the key performance are presented.**

***Keywords:* Batch service, Discrete-time queue, Departure epochs, slot, system occupancy and inter departure time distribution, vacation termination epochs.**

1. INTRODUCTION

The article deals with a vacation queueing model with bulk service rule .Using the discrete time analysis of a queue of packets of information in telecommunication networks, has been mode in the study of transportation systems. These queueing of packets can be model even when time is divided into equal intervals called slots. The numbers of customers that arrive one by one according to a follow geometric distribution, arriving packets first joint and wait in the queue called secondary queue before and move in batches of packets of minimum size number L and maximum size number K to the queue called primary queue after whenever the latter opens and service times of all customers are assumed to be a common follows geometric distribution. The ongoing service when $'j(L \leq j \leq K)'$and the service times are not affected by such inclusion.

In the queueing literature, exposes many useful results for queues with bulk service of a single server and queues with vacation. Relatively few results are available for queues that allow before and after both customer and sever vacation. A n extensive investigation on single server queueing system with vacation has been mode in recent years by many researchers. Miller (1964) is the first to study such a model, where the server is unavailable during some random length of time (referred to vacation) for the $M/G/1$ queueing system. Queue with vacation has been studied by number of authors Cooper (1970) and Doshi (1986). Harris and Marchal (1988) and Shanthikumar (1988) have proved the stochastic decomposition result for unfinished work in the system and additional delay due to the vacation time respectively in more general setting.

Krishna Reddy.*et.al.*, (1992) analyzes a queue with general bulk service and server vacation. Dhas.*et.al.*, (1992) considers a pre-emptive priority queue with general bulk service and vacation. Lee *et.al* (1996) derives the probability generating function of a fixed size batch service queue with server vacation. Lee. *et.al.*, (1997) establishes for the first time in their on batch service queueing system with vacation sever unavailability may reduce the waiting time in some batch service queue system. Kalyanaraman and Ayyapan (2000) analysed a single server vacation queue with balking and with single and batch service. They derived the steady probability distribution of the number of customers in the queue. Chinnadurai. *et.al*.,(2002) analyses the vacation queue with balking and with state dependent service rate.

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VeenaGoswami and VijiyaLaxmi (2011) discussed the inter-arrival times of customers are arbitrarily distributed, the service times and vacation times are both geometrically distributed. The server starts service in batches of maximum number size 'b' and minimum number threshold value 'a', otherwise if goes for vacation. There is a provision to include the late arriving customer in the ongoing service batch if the size of the batch being served less than a threshold value $'d(a \leq d \leq b)'$ and the service time is not affected by such inclusions. Recently, VijiyaLamxiet.al., (2013) studies discrete-time single server queue and the inter-arrival times of customer are assumed to be (i.i.d).The service time and service time during a vacation period follows geometrically distributed. Sree Parimalaand Palaniammal (2014) present the analytical solution of two servers bulk service queueing models in which only one server is allowed for vacation at a time to avoid the inconvenience to the customers.

In the analysis of the present study is continued under the assumption that service time of batches are independent of the number of packets in any batch and the simultaneous occurrence of a packet arriving and a vacation termination in a single slot is ruled out. The primary focus is on deriving various performance measures of the steady state distribution of the batch server at departure instants and also on numerical illustrations.

In the present investigation an attempt has been made to analyze the server's delayed and single vacation. This paper is organized as follows: Section 2 contains a brief motivation and description. In Section 3, the joint distribution of system occupancy at departure epoch is derived and inter-departure time and number of packets are discussed in Section 4. Numerical illustration in the form of tables and graphs are presented in Section 5. Finally, in section 6 concludes the paper.

2. MOTIVATION AND DESCRIPTION OF THE MODEL

Queueing system in which the server works on primary and secondary customers arise naturally (and sometimes not so naturally) as model of many computer, communication and production systems. The server working on the secondary customers is equivalent to the server taking a vacation and not being available to the primary custom as during this period. Thus, there is a natural interest in the study of queueing system with server vacation.

2.1. The arrival time distribution:

In this model geometric distribution and single vacation queue are considered. For geometric vacation the following mathematical definition is adopted. The server takes a vacation when the system becomes empty after a service or a vacation. However after each service ,if there are some customers in the queue the server may take a vacation with some probability 'p' or start a new service (if a customer is present) with probability $q(p+q=1)$. In this model we assume that the inter-arrival "A_n" of customers are independent and geometric distribution with probability mass function (p.m.f). $\{a(k) = \Pr(A_n = k): k = 0,1,2 \ldots.\}$,i.e.

$$a(k) = (1-p)^{k-1}p: \quad k = 0,1,2 \ldots. \qquad \ldots(1)$$

2.2. The mean number of customers in the inter-arrival time distribution:

The probability generating function for the number of customers in the system are obtained. The mean and variance of the number of customer in the system, waiting time of a customer in the system are also obtained. Some particular model are analyzed by assuming specific probability distribution to service time and vacation time

The probability generating function (p.g.f)

$$A(z) = \sum_{k=0}^{\infty} a(k)z^k \qquad = \sum_{k=0}^{\infty} pq^k q^{-1} z^r \qquad = \frac{p}{q}\sum_{k=0}^{\infty}(qz)^k = \frac{p}{q(1-qz)}$$

The first order derivative,

$$A'(z) = \frac{p}{q}(1-qz)^{-1} \qquad = \frac{p}{(1-qz)^2}$$

The second order derivative,

$$A''^{(z)} \quad = p(1-qz)^{-2} \qquad\qquad = \frac{2pq}{(1-qz)^3}$$

Hence,

$$E(A) = \frac{1}{p} \qquad \dots (2)$$

2.3. The variance of number of customers in the inter-arrival distribution:

From the previous calculation

$$= \frac{2q}{p^2} + \frac{1}{p} - \left(\frac{1}{p}\right)^2 \quad = \frac{q+1}{p^2} - \frac{1}{p^2}$$

Hence,

$$V(A) = \frac{q}{p^2} \qquad \dots (3)$$

2.4. The service time distribution:

The service time and vacation time are independently and identically distributed random variables. The customer are served in the order of their arrival in batches of size, lying in the closed interval (L,K). The server takes a vacation, if the number of customers waiting in queue is less than number of customers to wait in queue. when the server returns from vacation, if more then (L-1) or less than (K+1) customers are waiting then all the customers are taken into service. Customers, who arrive, while a group of customers are being served, cannot join the ongoing service even if some space is available. The service time for the n^{th} batch is of length B_n, where the B_n are random variables which follows geometric distribution with (p. m. f). $\{ b(k) = \Pr(B_n = k): k = 0,1,2 \dots . \}$,service time is independent of the number of slots in the batch.

$$B(k) = (1-p_1)^{k-1} p_1: \quad k = 0,1,2 \dots . \qquad \dots (4)$$

2.5. The mean number of customers in the service time distribution:

The probability generating function of service time distribution B (z) is given by

$$B(z) = \sum_{k=0}^{\infty} b(k) z^k \qquad = \sum_{k=0}^{\infty} (1-p_1)^{k-1} p_1 z^k \qquad = \frac{p_1}{q_1} \sum_{k=0}^{\infty} (q_1 z)^k$$

which implies that

$$B(z) = \frac{p_1}{q_1(1-q_1 z)}$$

The first order derivatives,

$$B'(z) = \frac{p_1}{q_1}(1-q_1 z)^{-1}$$

and second order derivatives,

$$B''(z) \quad = p_1(1-q_1 z)^{-2}$$

Hence,

$$E(A) = \frac{1}{p_1} \qquad \dots (5)$$

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2.6. The variance of number of customers in the service time distribution:

From the above result, variance can be write as

$$V(\boldsymbol{B}) = \frac{2q_1}{{p_1}^2} + \frac{1}{p_1} - \left(\frac{1}{p_1}\right)^2 = \frac{q_1}{{p_1}^2} \qquad \dots (6)$$

3. QUEUEING SYSTEM UTILIZATION

Utilization is the proportion of the system resources which is used by the traffic when arrives at it (Geometric arrivals and Geometric server), then it is given by the mean arrival rate over the mean service rate that is,

$$\rho = \frac{E(B)}{KE(A)} \qquad \dots (7)$$

The server load,

$$\rho = \frac{p}{Kp_1} < 1 \qquad \dots (8)$$

which is less than 1.

The probability of a packet arriving at the close of the slot is $'p'$ and it is not arriving at that point is $'q'$ which implies that the inter-arrival time geometrically distributed. A batch of packets of size $'j(L \leq j \leq K)$ starts service at the beginning of a slot, and may only end service just before the end of a slot. The probability of a batch being transported by the server at the end of a slot is probability "p_1" and the probability of the batch not being transported is "q_1". The probability that the server terminates its vacation when it is in vacation at the close of a slot is "p_2" and that of it not terminating the vacation at that point is "q_2" which implies that the vacation time is geometrically distributed. Service time of a batch is independent of the number of packets in a batch and simultaneous occurrence of both arrival and departure the vacation termination is a single slot are ruled out.

4. THE JOINT DISTRIBUTION OF SYSTEM OCCUPANCY AT DEPARTURE EPOCHS

Let $X_n \in \{0,1,2, \dots, \infty\}$ be the number of packets accumulated in the system queue and service just after the server has left with n^{th} batch. The steady state distribution $\{x(k) = \lim_{n\to\infty} x_n(k) = \Pr(X_n = k) : k = 0,1,2, \dots \ \infty\}$ of system occupancy at departure epochs is derived using the embedded Markov Chain technique.

Let g_n be random variable denoting the number of packets that reach the system during the n^{th} service. Then the joint distribution$\{g_n(k): k = 0,1,2, \dots\}$ of $g_n$can be derived as follows:

$$g(k) = \sum_{m=k+1}^{\infty} \binom{m-1}{k} \{p^k q^{m-1-k}\}\{p_1 {q_1}^{m-1}\};\ k = 0,1, \dots \qquad \dots (9)$$

For $k = 0,1,2, \dots$

$$g(k) = U(1-U)^k \quad where\ U = \frac{p_1}{p + p_1 - pp_1} \qquad \dots (10)$$

Putting $r = m - 1 - k$, so that $m - 1 = r + k$. Further on the limit $m = k + 1, r = 0$ and $m = \infty$ implie $r = \infty$. Then the above expression will be

$$g(k) = \sum_{m=k+1}^{\infty} \binom{m-1}{k} \{p^k (1-p)^{m-1-k}\}\{p_1 (1-p_1)^{m-1}\}$$

$$= \frac{p^k p_1 (1-p_1)^k}{\big(1-(1-p)(1-p_1)\big)^{k+1}} \quad where \sum_{r=o}^{\infty} \binom{r+k}{k} d^r = {}^1\!/_{(1-d)^{k+1}}$$

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$$= \frac{p^k p_1 (1-p_1)^k}{\left(1-\left((1-p)(1-p_1)\right)\right)^{k+1}} \qquad = \frac{p^k p_1 (1-p_1)^k}{\left(1-(1-p_1-p+pp_1)\right)^{k+1}}$$

$$g(k) = \frac{p^k p_1 (1-p_1)^k}{(p_1+p-pp_1)^{k+1}} = \left(\frac{p_1}{p_1+p-pp_1}\right)\left(1-\frac{p_1}{p_1+p-pp_1}\right)^k$$

Let H be random variable denoting the number of packets that reaching the system during a vacation period. Then the joint distribution $\{(k): k = 0,1,2,\dots\}$ of g_n can be derived as follows:

$$h(j) = \sum_{m=j+1}^{\infty} \binom{m-1}{j} \{p^j q^{m-1-j}\}\{p_2 {q_2}^{m-1}\};\, j = 0,1,\dots \qquad \dots(11)$$

For $k = 0,1,2,\dots\infty$

$$i.e, \quad h(j) = U_2(1-U_2)^j \quad where\ U_2 = \frac{p_2}{p+p_2-pp_2} \qquad \dots(12)$$

Putting $\mathrm{r = m-1-j}$, so that $\mathrm{m-1 = r+j}$. Further on the limit $\mathrm{m = j+1, r = 0}$ and $\mathrm{m} = \infty$ implies $\mathrm{r} = \infty$. Then the above expression will be

$$h(j) = \sum_{m=j+1}^{\infty} \binom{m-1}{j} \{p^j (1-p)^{m-1-j}\}\{p_2(1-p_2)^{m-1}\}$$

$$h(j) = \{p^j p_2 (1-p_2)^j\} \sum_{r=0}^{\infty} \binom{m-1}{j} \{p_2 (1-p)^r (1-p_2)^r\}$$

$$= \frac{p^j p_2 (1-p_2)^j}{\left(1-(1-p)(1-p_2)\right)^{j+1}} \quad where \sum_{r=o}^{\infty} \binom{r+j}{j} d^r = {}^{1}\!/_{(1-d)^{j+1}}$$

$$= \frac{p^j p_2 (1-p_2)^j}{\left(1-\left((1-p)(1-p_2)\right)\right)^{k+1}} \qquad = \frac{p^j p_2 (1-p_2)^j}{\left(1-(1-p_2-p+pp_2)\right)^{j+1}}$$

$$h(j) = \frac{p^j p_2 (1-p_2)^j}{(p_2+p-pp_2)^{j+1}} = \left(\frac{p_2}{p_2+p-pp_2}\right)\left(1-\frac{p_2}{p_2+p-pp_2}\right)^j$$

Let consider the basic stochastic process $\{\mathrm{X_n}\}$ which is a Markov Chain (MC) induced by the MC related to the M/M/k queue. The transition probability matrix of the Marko Chain is $\mathrm{P} = (\mathrm{p_{ij}})$.

where

$$\mathrm{p_{ij}} = \begin{cases} \sum_{n=0}^{\mathrm{b-i}} h_n \mathrm{g}_0 & ;0 \le i \le a-1 \quad and j \ge 0 \\ \sum_{n=0}^{\mathrm{b-i}} h_n \mathrm{g_j} + \sum_{n=\mathrm{b-i+1}}^{\mathrm{b+j-i}} h_n \mathrm{g_{b-i+j-n}} & ;0 \le i \le a-1 \quad and j \ge 0 \\ \mathrm{g_j} & ;a \le i \le b \ \ and j \ge 0 \\ g_{j-(i-b)} & ;i \ge (\mathrm{b}+1) and j \ge (\mathrm{i-b}) \ge 0 \\ 0 & ;\text{otherwise} \end{cases} \qquad \dots(13)$$

Let the unknown probability row vector $X_n = (X_0, X_1, X_2, \dots)$ represent solving the following system of equation.

$$X_n = X_n P,\ and\ X_n e = 1 \qquad \dots(14)$$

where"e" denotes the stands for the row vector of unit elements. A number of numerical using the method could be suggested to solve the system of equation (14). For example, an algorithm for solving (14) is given by Latouche and Ramaswami (1999).

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5. DISTRIBUTION OF OCCUPANCY JUST BEFORE BATCH DEPARTURE EPOCHS

The joint probability $P(A=j,V=m)=h(j,m)$ say that $'j'$ packets arrive during a vacation period cosisting of $'m'$ slot is

$$h(j,m)=\binom{m-1}{j}\{p^j(1-p)^{m-1-j}\}\{p_2(1-p_2)^{m-1}\};\quad j=0,1,\ldots.\ and\ m=j+1,j+2,\ldots \qquad \ldots(15)$$

Let the partial generating function of the vacation period, given that packets arrive during this vacation period be $H_j(z)$;

$$H_j(z)=\sum_{m=j+1}^{\infty}h(j,m)z^m=\sum_{m=j+1}^{\infty}\binom{m-1}{j}\{p^j(1-p)^{m-1-j}\}\{p_2(1-p_2)^{m-1}\}z^m;\ j=0,1,\ldots$$

$$=\left(\frac{p_2z}{1-qq_2z}\right)\left(\frac{pp_2z}{1-qq_2z}\right)^j \qquad \ldots(16)$$

It is observed that the following results can be obtained from (16) for $j=0,1,2,\ldots\infty$:

$$H_j(1)=h(j);$$

$$\sum_{j=0}^{\infty}H_j(z)=H(z)=\left(\frac{p_2z}{1-qq_2z}\right)$$

which is the probability generating function of $\{h(j);\ j=0,1,2,\ \ldots\infty\}$

$$H'_j(z)=\left(\frac{p_2}{1-qq_2z}\right)\left(\frac{pp_2z}{1-qq_2z}\right)^j\left(\frac{j+1}{1-qq_2z}\right)=h(j)\left(\frac{j+1}{p_2+pq_2}\right)$$

$$H'_j(z)=\frac{p_2(j+1)(pq_2z)^{j-1}(pq_2)\{j+2qq_2\}}{(1-qq_2z)^{j+z}}=H^jj(1)\left(\frac{j+2qq_2}{p_2+pq_2}\right)$$

Let Y_n random system occupancy immediately at the beginning of the n^{th} service period. As the respective distribution of Y_n is$\{y_n(j)=Pr(Y_n=j)\}$: $j=L,L+1,L+2,\ldots\}$,it could be seen that$\{y(j)=\log_{n\to\infty}y_n(j)=Pr(Y_n=j)\}$: $j=L,L+1,L+2,\ldots\}$, which is

$$Y_n(j)=P(Y_n=j)=\begin{cases}X(i)+\sum_{i=0}^{L-1}x & ;\ if X_{n-1}=j>L\\ X(L)+\sum_{i=0}^{L-i}X(i)\sum_{r=i}^{L}(r-i)hh; & if\ \ X_{n-1}\le j=L\end{cases} \qquad \ldots(17)$$

6. INTER-DEPARTURE TIME AND NUMBER OF PACKETS IN A BATCH

Let D_b random varible denote the time number of slots between the departures of two consecutive batches of packets. The inter-departure time consists of the inter-arrival time of the (L-X) packets needed to make up a batch and the service time of that batch. Hence, the distribution $\{d_b(k)=Pr(D_b=k)\}$ of inter-departure time D_b is

$$D_b(k)=b(k)\sum_{i=L}^{\infty}x(i)+\sum_{i=0}^{L-1}\left\{\sum_{j=L}^{\infty}h_{j-i}()^*b(k)+\sum_{j=i}^{L-1}h_{j-i}()^*a^{*(L-j)}()^*b(k)\right\}x(i) \qquad \ldots(18)$$

where * stands for convolution operation and a^{*i} denotes i-fold convolution of a(k) with itself . The probability generating function of D_b is

$$D_b(z)=B(k)\left[\sum_{i=L}^{\infty}x(i)+\sum_{i=0}^{L-1}\left\{\sum_{j=L}^{\infty}H_{j-i}(z)+\sum_{j=i}^{L-1}H_{j-i}(z)\{A(z)\}^{(L-j)}\right\}x(i)\right]$$

The first order derivative,

$$D_b'(z) = B'(z)\left[\sum_{i=L}^{\infty} x(i) + \sum_{i=0}^{L-1}\left\{\sum_{j=L}^{\infty} H_{j-i}(z) + \sum_{j=i}^{L-1} H_{j-i}(z)\{A(z)\}^{(L-j)}\right\}x(i)\right]$$

$$+B(z)\left[\sum_{i=0}^{L-1}\left\{\sum_{j=L}^{\infty} H_{j-1}'(z) + \sum_{j=1}^{L-1}\{H_{j-1}'(z)\}\{A(z)\}^{(L-j)} + H_{j-1}(z) + (L-j)\{A(z)\}^{(L-j-1)}A'^{(z)}\right\}x(i)\right]$$

The second derivative,

$$D_b''(z) = B''(z)\left[\sum_{i=L}^{\infty} x(i) + \sum_{i=0}^{L-1}\left\{\sum_{j=L}^{\infty} H_{j-i}(z) + \sum_{j=i}^{L-1} H_{j-i}(z)\{A(z)\}^{(L-j)}\right\}x(i)\right]$$

$$+B'(z)\left[\sum_{i=0}^{L-1}\left\{\sum_{j=L}^{\infty} H'_{j-1}(z) + \sum_{j=i}^{L-1}\{H'_{j-i}(z)\{A(z)\}^{(L-j)} + H_{j-i}(z)(L-j)\{A(z)\}^{(L-j-i)}A'(z)\}\right\}x(i)\right]$$

$$+B(z)\left[\sum_{i=0}^{L-1}\left\{\sum_{j=L}^{\infty} H''_{j-1}(z)\right.\right.$$

$$+\sum_{j=i}^{L-1}\Big\{H''_{j-i}(z)\{A(z)\}^{(L-j)} + H'_{j-i}(z)(L-j)\big(A(z)\big)^{(L-j-i)}A'^{(z)} + H'_{j-i}(z)(L-j)\big(A(z)\big)^{(L-j-i)}A'^{(z)}$$

$$\left.\left.+H_{j-i}(z)(L-j)\left\{(L-j-1)A^{(L-j-2)}(z)\big(A'(z)\big)^2 + A(z)^{(L-j-1)}A''(z)\right\}\Big\}\right\}x(i)\right]$$

$$+B'(z)\left[\sum_{i=0}^{L-1}\left\{\sum_{j=L}^{\infty} H'_{j-1}(z) + \sum_{j=i}^{L-1}\{H'_{j-i}(z)\{A(z)\}^{(L-j)} + H_{j-i}(z)(L-j)\{A(z)\}^{(L-j-i)}A'(z)\}\right\}x(i)\right]$$

and thus

$$E(D) = D'_b(1) + \left[\sum_{i=o}^{L-1}\left\{H'(1) + \sum_{j=i}^{L-1} H_{j-i}(1)(L-j)A'^{(1)}\right\}x(i)\right]$$

$$= E(B) + E(V)\sum_{i=0}^{L-1} x(i) + \sum_{i=0}^{L-1}\left[\left\{\left(\frac{p_2}{p_2+pq_2}\right)(L-i)E(A) + \sum_{j=i+1}^{L-1}\left(\frac{p_2}{p_2+pq_2}\right)\left(\frac{pq_2}{p_2+pq_2}\right)^{j-i}(L-j)E(A)\right\}x(i)\right]$$

where

$$H' = E(V) = \left(\frac{1}{p_2}\right)$$

$$+B(1)\left[\sum_{i=0}^{L-1}\left[H''(1)\right.\right.$$

$$+\sum_{j=i}^{L-1}\{H'_{j-1}(1)(L-j)E(A) + H_{j-i}(1)(L-j)\{(L-j-1)A'(1)^2 + A''(1)\} + H'_{j-1}(1)(L-j)E(A)$$

$$\left.\left.+H_{j-i}(1)(Lj)\{(L-j-1)A'(1)^2 + A''(1)\}\}\right]x(i)\right]$$

Let S be a random variable denoting the number of lots in a batch at the beginning of a batch service and $s(k) = Pr(S = k)$for $k = L, L+1, \dots, K$

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$$S(j) = \begin{cases} y(L), & j = L \\ y(j), & L < j < k \\ \sum_{r=k}^{\infty} y(r), & j = k \end{cases} = \begin{cases} x(L) + \sum_{i=0}^{L-1} x(i) \sum_{r=i}^{L} h(r-j), j = L \\ x(j) + \sum_{i=0}^{L-1} x(i) h(j-i), L < j < 1 \\ \sum_{i=k}^{\infty} x(j) + \sum_{i=0}^{L-1} x(i) \sum_{r=k}^{\infty} h(r-i), j = K \end{cases}$$

The joint distribution $\{f(k,j) = Pr(D_b = k,\ S = j): k \geq 1, L \leq j \leq K\} D_b$ and S is

$$f(k,j) = \partial(k,j)\left[b(k)\sum_{i=k}^{\infty} x(i) + \sum_{i=0}^{L-i} x(i) \sum_{r=k}^{\infty} h_{r-i} * b(k)\right] + \partial(L,j)\left[x(L)b(k) + \sum_{i=0}^{L-i} x(i)\left\{h_{L-i} * b(k) + \sum_{i=0}^{L-i} h_{j-i} * a^{L-j}(\,)^{*} b(k)\right\}\right] + [1 - \partial(k-j) - \partial(L,j)]\left[b(k)x(j) + \sum_{i=0}^{L-1}\{h_{j-i} b(k)\} x(i)\right] \quad \ldots (19)$$

where$\partial(\mathrm{i},\mathrm{j}) = 1$ when $\mathrm{i} = \mathrm{j}$ and 0 otherwise

The expectation of the random variables D_b,S and their product D_b,S given below.

$$E(D_b,S) = E(S)E(B) + K\left\{\sum_{i=0}^{L-i} x(i) \sum_{r=k}^{\infty} H'_{r-i}(1)\right\} + L\sum_{i=0}^{L-i} x(i)\left\{H'_{L-i}(1) + \sum_{j=i}^{L-1}\{H'_{j-i}(1) + h_{j-i}(L-j)E(A)\}\right\} + \left(\sum_{j=L+1}^{K-1} j\right)\left\{\sum_{i=0}^{L-1} H'_{j-i}(1) x(i)\right\} \quad \ldots (20)$$

In the similar manner the variance of D_b and S can be calculated. To know how strongly these two random variables depend on each other, co-efficient of correlation $'r_1'$ can be compute and it can be defined as

$$r_1 = \frac{E(D_b,S) - E(D_b)E(S)}{\sqrt{Var(D_b)}\sqrt{Var(S)}} \quad \ldots (21)$$

Similarly Let S_2 random variables denoting the number of lots in a batch just before the batch $\{s_2(k) = \Pr(S_2 = k)\ :\ k = L, L+1, L+2, \ldots K\}$

$$s_2(j) = \begin{cases} \sum_{j=k}^{\infty} g(j) & j = K \\ g(j) & L \leq j < K \end{cases} \quad \ldots (22)$$

The moment of S_2 can be calculated from (22).By using (15) in one could rewrite (16) in terms of g (.) values as we get

$$d_b(k) = b(k)\sum_{i=L}^{\infty} g(i+K) + \sum_{i=0}^{L-1}[a^{*(L-i)} * b(k)] g(i+K) + (a^{*L} * b(k)) \sum_{j=L}^{K-1} g(j) \quad \ldots (23)$$

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We derive joint distribution of D_s and S_2 the usual arguments lead to the following difference- differential equation $\{g(k,j) = \Pr(D_b = k, S_2 = j) : k \geq 1, L \leq j \leq K\}$

$$g(k,j) = \partial(K,j)\left(b(k)\sum_{i=L}^{\infty} g(i+K) + \sum_{i=0}^{L-1}[a^{*(L-i)} * b(k)]g(i+K)\right) + (1-\partial(K,j))(a^{*L} * b(k))\, g(j) \qquad \dots(24)$$

Thus,

$$E(D_{b,}S_2) = K\left(E(B)\sum_{i=L}^{\infty} g(i+K) + \sum_{i=0}^{L-1}[(L-i)E(A)+E(B)]g(i+k)\right) + (LE(A)+E(B))\sum_{j=L}^{K-1} j\, g(j) \qquad \dots(25)$$

As in (17), one can calculate the correlation co-efficient, say r_2 , between D_b and S_2 .

$$r_2 = \frac{E(D_{b,}S_2) - E(D_b)E(S_2)}{\sigma_{D_b}\sigma_{S_2}} \qquad \dots(26)$$

7. NUMERICAL ILLUSTRATION

Consider the values p=0.98, p=0.45 ,p_2=0.28,$\alpha = 1$ and K=20 be considered so that $\rho = 0.1089$. For $L = 1, 2, \dots 20(= K)$, below shows the numerical values of the mean, variance and the co-efficient of correlation r (r2)of the stationary queue under study. In the results obtained on $\{x(i),\ i = 0,1, \dots \infty\}$ sum over 'i' is truncated at 150 and been shown in table-1 below.

Table-1: Departure epoch measures

L	$E(D_b)$	$Var(D_b)$	E(S)	Var(S)	$E(S_2)$	$V(S_2)$	r_1	r_2
1	6.077	10.647	2.802	7.518	4.00	10.05	.85590	-.01019
2	6.575	9.102	3.290	6.015	4.49	8.55	.82839	-.00980
3	7.222	7.671	3.923	4.617	5.12	7.15	.79134	-.01025
4	7.974	6.484	4.661	3.453	5.86	5.99	.74543	-.01126
5	8.803	5.554	5.472	2.535	6.67	5.08	.69172	-.01260
6	9.686	4.851	6.338	1.836	7.53	4.38	.63200	-.01408
7	10.607	4.332	7.241	1.315	8.44	3.86	.56859	-.01560
8	11.557	3.957	8.172	.932	9.37	3.48	.50408	-.01719
9	12.526	3.690	9.122	.653	10.32	3.19	.44099	-.01910
0	13.509	3.505	10.087	.453	11.28	2.97	.38108	-.02186
11	14.501	3.382	11.061	.309	12.25	2.80	.32563	-.02629
12	15.499	3.308	12.043	.208	13.23	2.64	.27521	-.03371
13	16.498	3.277	13.030	.136	14.21	2.48	.22972	-.04579
14	17.495	3.291	14.021	.087	15.18	2.28	.18883	-.06489
15	18.482	3.360	15.014	.053	16.15	2.03	.15169	-.09349
16	19.449	3.512	16.010	.031	17.10	1.68	.11733	-.1337
17	20.376	3.794	17.006	.016	18.01	1.24	.08521	-.18503
18	21.229	4.276	18.004	.007	18.84	.73	.05385	-.23976
19	21.945	5.018	19.002	.002	19.55	.25	.02199	-.27140
20	22.408	5.898	20.000	.000	20.00	.00	.00000	.00000

The r_1 and r_2 value is zero when L=K= 20 as the batch size is constant in this specific case. For L=1 to 20, values of the co-efficient of correlation r_1 and r_2 are negative. It indicates long inter-departure times correspond to small number in system occupancy at departure epochs to form a batch at the beginning of their service epoch. Mean values of both random variables D_b and var (D_b) increase with increase in L. But the variance values of the random variables decrease with the increase with the increase in L values as the range value between L and K becomes smaller and smaller value.

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7.1. Performance measure and cost function:

Cost function involving random variables D_b and S (S_2) can be formulated. Different cost values incurred by the coefficient of variation (CV) of S and by the coefficient of determination (CD) of r_1 can be defined as:

$\$c_1$ = Cos due to CV(S) (Cost due to CV S (S_2))

$\$c_2$ = Cos due to CV(r_1) (Cost due to CV S (r_2))

The total expected cost (TEC) function TEC_1 and TEC_2 say, subject to these costs depend on the parameters $p, p_{1,} L, K$ of the batch service system in terms of the pairs of random variables $(D_b, S)$and (D_b, S_2) and respectively they are

$TEC1(p, p_1, p_2, L, K) = C_1 CV(S) + C_2 r_1^2$

$TEC2(p, p_1, p_2, L, K) = C_1 CV(S_2){+}C_2 r_2^2$

TEC_1 and TEC_2 can be shown as convex functions, as L increases, CV(S) and $CV(S_2)$ decrease and r_1^2 and r_2^2 increase. Since the closed form of expression is not available for TEC, one can locate only local minimum in specific cases. An attempt has been made to trace it with the following assigned values:

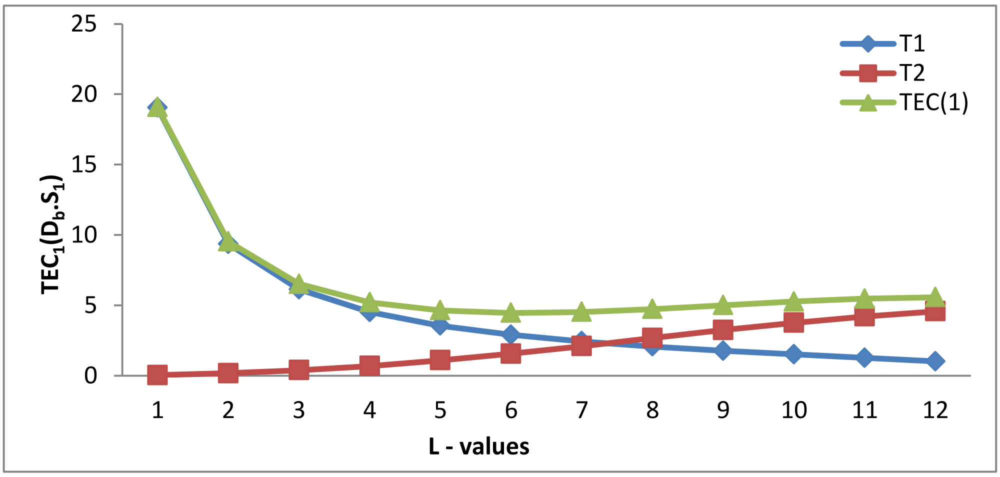
$c_1 = \$175.00, c_2 = \$\,452.00, p_1 = 0.950, p_2 = 0.470, \alpha = 3$andK=15so that $\rho = 0.404255$.

Table - 2: Cost function values

L	c_1 CV(S)	c_2 r_1^2	TEC_1	c_1 CV(S_2)	c_2 r_2^2	TEC_2
1	19.04	0.04	19.079	106.39	0	106.398
2	9.36	0.17	9.528	85.6	0.03	85.628
3	6.13	0.39	6.519	71.32	0.09	71.406
4	4.52	0.69	5.214	60.75	0.25	61.008
5	3.55	1.09	4.638	52.46	0.63	53.095
6	2.9	1.56	**4.462**	45.6	1.44	47.031
7	2.43	2.1	4.529	39.6	3.03	42.636
8	2.07	2.67	4.738	34.11	5.95	40.061
9	1.77	3.24	5.009	28.86	10.75	**39.606**
10	1.51	3.76	5.266	23.67	17.66	41.328
11	1.26	4.21	5.473	18.44	25.86	44.304
12	1.01	4.57	5.573	13.22	33.71	45.925

Fig - 1.Graph of TEC_1

TEC curve: D_b Vs S_1 c_1=$175.00, c_2=$452.00,p=0.950, p_1=0.470, α=3 and K=15 so that ρ=0.404255



An attempt has been made to trace it with the following pre-assigned values: c_1=$175.00,$c_2$=$452.00,p=0.950,p_1=0.470, α=3 and K=15 so that ρ=0.404255. Based on the corresponding results of TEC_1/TEC_2, graphs have been drawn and presented in Fig-1 and Fig-2 respectively where CV=(S.D/Mean) only.

TEC curve: Db Vs S_2: c_1=$175.00,$c_2$=$452.00,p=0.950,p_1=0.470, α=3 and K=15 so that ρ=0.404255

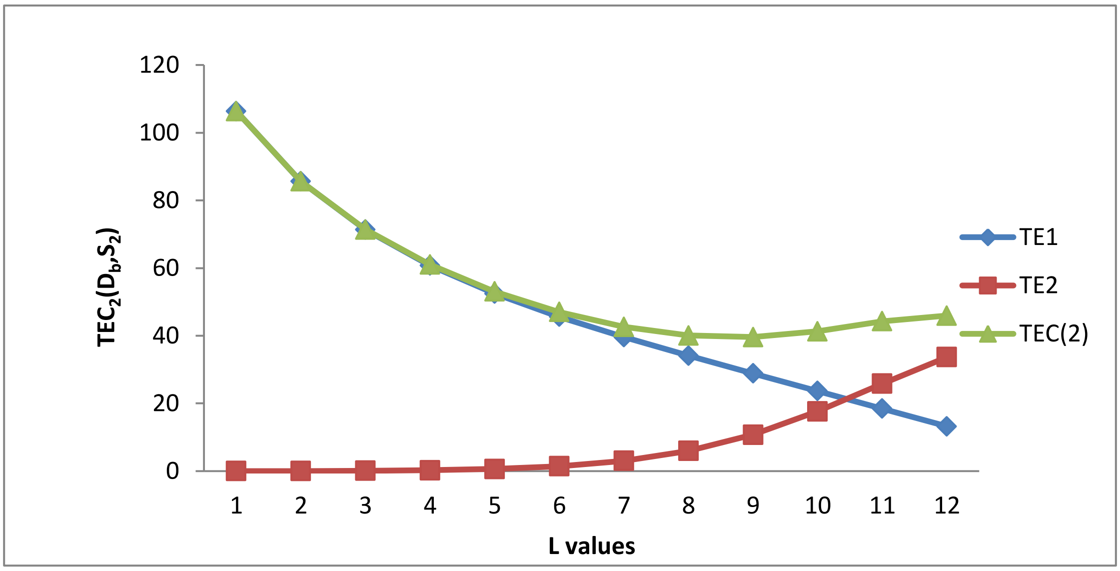


Fig - 2: Graph of TEC_2

The corresponding results of TEC_1/TEC_2 are presented in table-2 where CV= (S.D/Mean) only. It can be noticed that local minimum TEC_1 occurs when L = 6 and TEC_2 does when the local minimum is L = 9. Based on this information regarding local minimum on 'L' ,the facility rendering batch service with the rule (L,K) can call the server (the transportation vehicle from outside) to come to the service point and thereby saving the amount of transportation cost, which is the primary objective in transportation and optimization problems.

8. CONCLUSION

The basic analysis relating to the moments of queue contents of a vacation queueing model with bulk service rule is well-discussed. There are two queues, separated by that arriving packets first join and wait in the secondary queue before and more in batches of minimum size number L packets and maximum size number K to the primary queue after whenever the latter opens. This happens at the end of the last slot of a vacation period and is then served in batches $'j(L \leq J \leq K)'$ before leaving the primary queue. Both primary and secondary queue have an infinite capacity. Various performance measures of the steady state distribution of the batch server have been obtained numerically and are provided.

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